Moments - Questions

June 2017 Mathematics Advanced Paper 1: Mechanics 1

1.

3. A plank AB has length 6 m and mass 30 kg. The point C is on the plank with $CB = 2$ m. The plank rests in equilibrium in a horizontal position on supports at A and C. Two people, each of mass 75 kg, stand on the plank. One person stands at the point P of the plank, where $AP = x$ metres, and the other person stands at the point Q of the plank, where $AQ = 2x$ metres. The plank remains horizontal and in equilibrium with the magnitude of the reaction at C five times the magnitude of the reaction at A. The plank is modelled as a uniform rod and each person is modelled as a particle.

    (*a*) Find the value of x.

(7)

    (*b*) State two ways in which you have used the assumptions made in modelling the plank as a uniform rod.
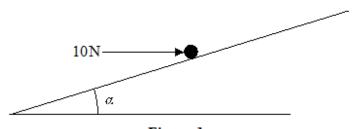
(2)

2.

4.

Figure 1

A particle P of mass 5 kg is held at rest in equilibrium on a rough inclined plane by a horizontal force of magnitude 10 N. The plane is inclined to the horizontal at an angle α where $\tan \alpha = \dfrac{3}{4}$, as shown in Figure 1. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ. Given that P is on the point of sliding down the plane, find the value of μ.
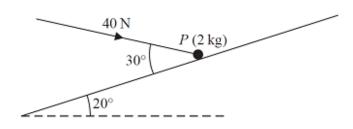
(9)

3.

5.

Figure 2

A particle P of mass 2 kg is held at rest in equilibrium on a rough plane by a constant force of magnitude 40 N. The direction of the force is inclined to the plane at an angle of 30°. The plane is inclined to the horizontal at an angle of 20°, as shown in Figure 2. The line of action of the force lies in the vertical plane containing P and a line of greatest slope of the plane. The coefficient of friction between P and the plane is μ.

Given that P is on the point of sliding up the plane, find the value of μ.

(10)

4.

6. A non-uniform plank AB has length 6 m and mass 30 kg. The plank rests in equilibrium in a horizontal position on supports at the points S and T of the plank where $AS = 0.5$ m and $TB = 2$ m.

When a block of mass M kg is placed on the plank at A, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about S.

When the block is moved to B, the plank remains horizontal and in equilibrium and the plank is on the point of tilting about T.

The distance of the centre of mass of the plank from A is d metres. The block is modelled as a particle and the plank is modelled as a non-uniform rod. Find

(i) the value of d,
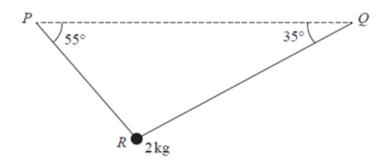
(ii) the value of M.

(7)

5.

3.

Figure 1

A particle of mass 2 kg is suspended from a horizontal ceiling by two light inextensible strings, PR and QR. The particle hangs at R in equilibrium, with the strings in a vertical plane. The string PR is inclined at $55°$ to the horizontal and the string QR is inclined at $35°$ to the horizontal, as shown in Figure 1.

Find

(i)   the tension in the string PR,

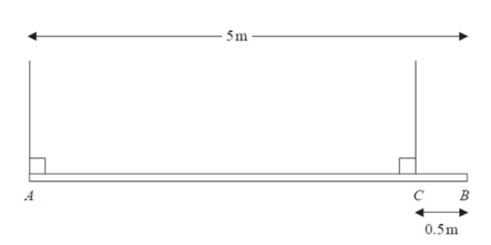(ii) the tension in the string QR.

(7)

6.

5.

Figure 3

A beam AB has length 5 m and mass 25 kg. The beam is suspended in equilibrium in a horizontal position by two vertical ropes. One rope is attached to the beam at A and the other rope is attached to the point C on the beam where $CB = 0.5$ m, as shown in Figure 3. A particle P of mass 60 kg is attached to the beam at B and the beam remains in equilibrium in a horizontal position. The beam is modelled as a uniform rod and the ropes are modelled as light strings.

(a) Find

    (i) the tension in the rope attached to the beam at A,

    (ii) the tension in the rope attached to the beam at C.

(6)

Particle P is removed and replaced by a particle Q of mass M kg at B. Given that the beam remains in equilibrium in a horizontal position,

(b) find

    (i) the greatest possible value of M,

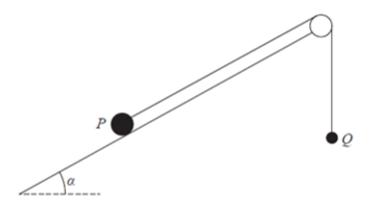   (ii) the greatest possible tension in the rope attached to the beam at C.

(6)

7.

8.

Figure 4

Two particles P and Q have mass 4 kg and 0.5 kg respectively. The particles are attached to the ends of a light inextensible string. Particle P is held at rest on a fixed rough plane, which is inclined to the horizontal at an angle α where $\tan \alpha = \dfrac{4}{3}$. The coefficient of friction between P and the plane is 0.5. The string lies along the plane and passes over a small smooth light pulley which is fixed at the top of the plane. Particle Q hangs freely at rest vertically below the pulley. The string lies in the vertical plane which contains the pulley and a line of greatest slope of the inclined plane, as shown in Figure 4. Particle P is released from rest with the string taut and slides down the plane.
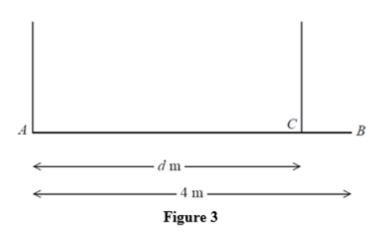
Given that Q has not hit the pulley, find

(a) the tension in the string during the motion,

(11)

(b) the magnitude of the resultant force exerted by the string on the pulley.

(4)

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8.

4.

Figure 3

A beam AB has weight W newtons and length 4 m. The beam is held in equilibrium in a horizontal position by two vertical ropes attached to the beam. One rope is attached to A and the other rope is attached to the point C on the beam, where $AC = d$ metres, as shown in Figure 3. The beam is modelled as a uniform rod and the ropes as light inextensible strings. The tension in the rope attached at C is double the tension in the rope attached at A.

(a) Find the value of d.

(6)

A small load of weight kW newtons is attached to the beam at B. The beam remains in equilibrium in a horizontal position. The load is modelled as a particle. The tension in the rope attached at C is now four times the tension in the rope attached at A.

(b) Find the value of k.

(6)

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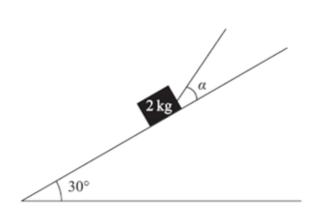
9.

3.

Figure 1

A box of mass 2 kg is held in equilibrium on a fixed rough inclined plane by a rope. The rope lies in a vertical plane containing a line of greatest slope of the inclined plane. The rope is inclined to the plane at an angle α, where $\tan\alpha = \frac{3}{4}$, and the plane is at an angle of 30° to the horizontal, as shown in Figure 1. The coefficient of friction between the box and the inclined plane is $\frac{1}{3}$ and the box is on the point of slipping up the plane. By modelling the box as a particle and the rope as a light inextensible string, find the tension in the rope.

(8)

10.

6. A beam AB has length 15 m. The beam rests horizontally in equilibrium on two smooth supports at the points P and Q, where $AP = 2$ m and $QB = 3$ m. When a child of mass 50 kg stands on the beam at A, the beam remains in equilibrium and is on the point of tilting about P. When the same child of mass 50 kg stands on the beam at B, the beam remains in equilibrium and is on the point of tilting about Q. The child is modelled as a particle and the beam is modelled as a non-uniform rod.

(a) (i) Find the mass of the beam.

(ii) Find the distance of the centre of mass of the beam from A.

(8)

When the child stands at the point X on the beam, it remains horizontal and in equilibrium. Given that the reactions at the two supports are equal in magnitude,

(b) find AX.

(6)

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11.

2. A steel girder AB, of mass 200 kg and length 12 m, rests horizontally in equilibrium on two smooth supports at C and at D, where $AC = 2$ m and $DB = 2$ m. A man of mass 80 kg stands on the girder at the point P, where $AP = 4$ m, as shown in Figure 1.
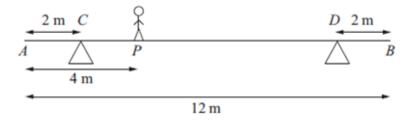

Figure 1

The man is modelled as a particle and the girder is modelled as a uniform rod.

(a) Find the magnitude of the reaction on the girder at the support at C.

(3)

The support at D is now moved to the point X on the girder, where $XB = x$ metres. The man remains on the girder at P, as shown in Figure 2.
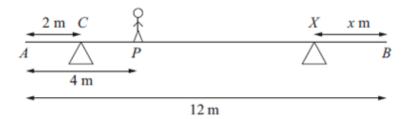

Figure 2

Given that the magnitudes of the reactions at the two supports are now equal and that the girder again rests horizontally in equilibrium, find

(b) the magnitude of the reaction at the support at X,

(2)

(c) the value of x.

(4)

12.

3. A particle P of mass 2 kg is attached to one end of a light string, the other end of which is attached to a fixed point O. The particle is held in equilibrium, with OP at 30° to the downward vertical, by a force of magnitude F newtons. The force acts in the same vertical plane as the string and acts at an angle of 30° to the horizontal, as shown in Figure 3.
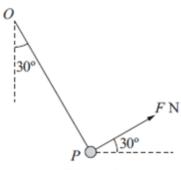

Figure 3

Find

(i) the value of F,

(ii) the tension in the string.

(8)

13.

4. A lifeboat slides down a straight ramp inclined at an angle of 15° to the horizontal. The lifeboat has mass 800 kg and the length of the ramp is 50 m. The lifeboat is released from rest at the top of the ramp and is moving with a speed of 12.6 m s⁻¹ when it reaches the end of the ramp. By modelling the lifeboat as a particle and the ramp as a rough inclined plane, find the coefficient of friction between the lifeboat and the ramp.

(9)
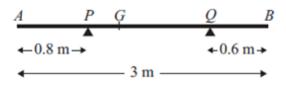
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14.

2.

Figure 1

A non-uniform rod AB has length 3 m and mass 4.5 kg. The rod rests in equilibrium, in a horizontal position, on two smooth supports at P and at Q, where $AP = 0.8$ m and $QB = 0.6$ m, as shown in Figure 1. The centre of mass of the rod is at G. Given that the magnitude of the reaction of the support at P on the rod is twice the magnitude of the reaction of the support at Q on the rod, find

(a) the magnitude of the reaction of the support at Q on the rod,

(3)
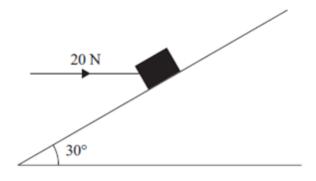
(b) the distance AG.

(4)

15.

3.

Figure 2

A box of mass 5 kg lies on a rough plane inclined at 30° to the horizontal. The box is held in equilibrium by a horizontal force of magnitude 20 N, as shown in Figure 2. The force acts in a vertical plane containing a line of greatest slope of the inclined plane.

The box is in equilibrium and on the point of moving down the plane. The box is modelled as a particle.

Find

(a) the magnitude of the normal reaction of the plane on the box,

(4)

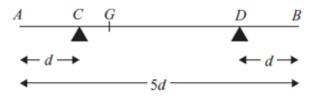(b) the coefficient of friction between the box and the plane.

(5)

16.

4.

Figure 1

A non-uniform rod AB, of mass m and length $5d$, rests horizontally in equilibrium on two supports at C and D, where $AC = DB = d$, as shown in Figure 1. The centre of mass of the rod is at the point G. A particle of mass $\frac{5}{2}m$ is placed on the rod at B and the rod is on the point of tipping about D.

(a) Show that $GD = \frac{5}{2}d$.

(4)

The particle is moved from B to the mid-point of the rod and the rod remains in equilibrium.

(b) Find the magnitude of the normal reaction between the support at D and the rod.
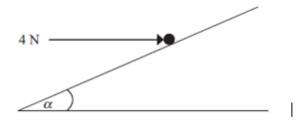
(5)

17.

3.

Figure 1

A particle of weight W newtons is held in equilibrium on a rough inclined plane by a horizontal force of magnitude 4 N. The force acts in a vertical plane containing a line of greatest slope of the inclined plane. The plane is inclined to the horizontal at an angle α, where $\tan \alpha = \frac{3}{4}$ as shown in Figure 1.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

Given that the particle is on the point of sliding down the plane,

(i) show that the magnitude of the normal reaction between the particle and the plane is 20 N,

(ii) find the value of W.

(9)

18.

5. A plank PQR, of length 8 m and mass 20 kg, is in equilibrium in a horizontal position on two supports at P and Q, where $PQ = 6$ m.

A child of mass 40 kg stands on the plank at a distance of 2 m from P and a block of mass M kg is placed on the plank at the end R. The plank remains horizontal and in equilibrium. The force exerted on the plank by the support at P is equal to the force exerted on the plank by the support at Q.

By modelling the plank as a uniform rod, and the child and the block as particles,

(a) (i) find the magnitude of the force exerted on the plank by the support at P,

(ii) find the value of M.

(10)

(b) State how, in your calculations, you have used the fact that the child and the block can be modelled as particles.
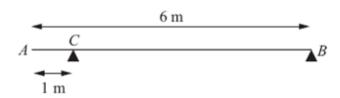
(1)

19.

3.

Figure 1

A uniform beam AB has mass 20 kg and length 6 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at C, where $AC = 1$ m, and the other is at the end B, as shown in Figure 1. The beam is modelled as a rod.

(*a*) Find the magnitudes of the reactions on the beam at B and at C.

(5)

A boy of mass 30 kg stands on the beam at the point D. The beam remains in equilibrium. The magnitudes of the reactions on the beam at B and at C are now equal. The boy is modelled as a particle.
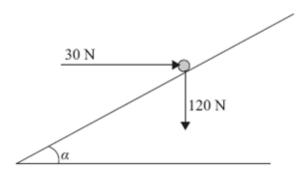
(*b*) Find the distance AD.

(5)

20.

6.

Figure 4

A particle of weight 120 N is placed on a fixed rough plane which is inclined at an angle α to the horizontal, where $\tan \alpha = \frac{3}{4}$.

The coefficient of friction between the particle and the plane is $\frac{1}{2}$.

The particle is held at rest in equilibrium by a horizontal force of magnitude 30 N, which acts in the vertical plane containing the line of greatest slope of the plane through the particle, as shown in Figure 2.

(a) Show that the normal reaction between the particle and the plane has magnitude 114 N.
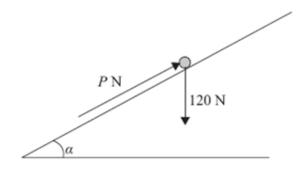
(4)

Figure 3

The horizontal force is removed and replaced by a force of magnitude P newtons acting up the slope along the line of greatest slope of the plane through the particle, as shown in Figure 3. The particle remains in equilibrium.

(b) Find the greatest possible value of P.

(8)

(c) Find the magnitude and direction of the frictional force acting on the particle when $P = 30$.

(3)

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21.

4. A beam AB has length 6 m and weight 200 N. The beam rests in a horizontal position on two supports at the points C and D, where $AC = 1$ m and $DB = 1$ m. Two children, Sophie and Tom, each of weight 500 N, stand on the beam with Sophie standing twice as far from the end B as Tom. The beam remains horizontal and in equilibrium and the magnitude of the reaction at D is three times the magnitude of the reaction at C. By modelling the beam as a uniform rod and the two children as particles, find how far Tom is standing from the end B.
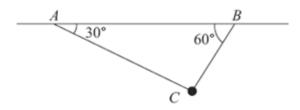
(7)

22.

3.

Figure 1

A particle of mass m kg is attached at C to two light inextensible strings AC and BC. The other ends of the strings are attached to fixed points A and B on a horizontal ceiling. The particle hangs in equilibrium with AC and BC inclined to the horizontal at 30° and 60° respectively, as shown in Figure 1.

Given that the tension in AC is 20 N, find

(*a*) the tension in BC,

(4)
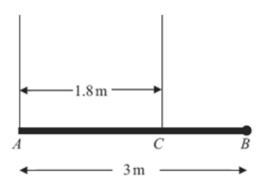
(*b*) the value of m.

(4)

23.

4.

Figure 2

A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at C is $\left(\dfrac{5}{6}W + \dfrac{100}{3} \right)$ N.

(4)

(b) Find, in terms of W, the tension in the rope attached to the pole at A.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of W.

(3)